$$\begin{bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} (X_u + X_{T_u}) & X_\alpha & X_q & -g\cos\theta_1 \\ \frac{Z_u}{U_1 - Z_{\dot{\alpha}}} & \frac{Z_\alpha}{U_1 - Z_{\dot{\alpha}}} & \frac{(Z_q + U_1)}{U_1 - Z_{\dot{\alpha}}} & \frac{-g\sin\theta_1}{U_1 - Z_{\dot{\alpha}}} \\ \frac{M_{\dot{\alpha}} Z_u}{U_1 - Z_{\dot{\alpha}}} + (M_u + M_{T_u}) & \frac{M_{\dot{\alpha}} Z_\alpha}{U_1 - Z_{\dot{\alpha}}} + (M_\alpha + M_{T_\alpha}) & \frac{M_{\dot{\alpha}}(Z_q + U_1)}{U_1 - Z_{\dot{\alpha}}} + M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ \frac{Z_{\delta_e}}{U_1 - Z_{\dot{\alpha}}} \\ \frac{M_{\dot{\alpha}} Z_{\delta_e}}{U_1 - Z_{\dot{\alpha}}} + M_{\delta_e} \\ 0 \end{bmatrix} \delta_e$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{U_1} & Y_p & Y_r - U_1 & g\cos\theta_0 & 0 \\ \frac{L_\beta + A_1[N_\beta + N_{T\beta}]}{(1 - A_1 B_1)U_1} & \frac{L_p + A_1 N_p}{1 - A_1 B_1} & \frac{L_r + A_1 N_r}{1 - A_1 B_1} & 0 & 0 \\ \frac{B_1 L_\beta + N_\beta + N_{T\beta}}{(1 - A_1 B_1)U_1} & \frac{B_1 L_p + N_p}{1 - A_1 B_1} & \frac{B_1 L_r + N_r}{1 - A_1 B_1} & 0 & 0 \\ 0 & 1 & \tan\theta_0 & 0 & 0 \\ 0 & 0 & \frac{1}{\cos\theta_0} & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \\ \psi \end{bmatrix} + \begin{bmatrix} Y_{\delta_A} & Y_{\delta_r} \\ \frac{L_{\delta_A} + A_1 N_{\delta_A}}{1 - A_1 B_1} & \frac{L_{\delta_r} + A_1 N_{\delta_r}}{1 - A_1 B_1} \\ \frac{B_1 L_{\delta_A} + N_{\delta_A}}{1 - A_1 B_1} & \frac{B_1 L_{\delta_r} + N_{\delta_r}}{1 - A_1 B_1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

Estabilidad y Control Detallado

Estabilidad Dinámica

Tema 14.5

Sergio Esteban Roncero
Departamento de Ingeniería Aeroespacial
Y Mecánica de Fluidos

Estabilidad Dinámica Longitudinal

- Estabilidad dinámica está presente si el movimiento dinámico del avión regresa eventualmente a su estado original.
- En el movimiento longitudinal se definen claramente dos modos:
  - Modo Fugoide (*Phugoid mode*)
    - $\alpha \approx$ cte
  - Modo de periodo corto (*Short Period*)
    - velocidad $\approx$ cte
- Amortiguamiento del modo de corto periodo:
  - Suave a alta velocidad y enérgico a baja.
- Amortiguamiento del modo fugoide difícil de concretar en diseño preliminar.
- El control es fundamental a baja velocidad para tener capacidad de rotación en despegue y maniobra en aproximación.

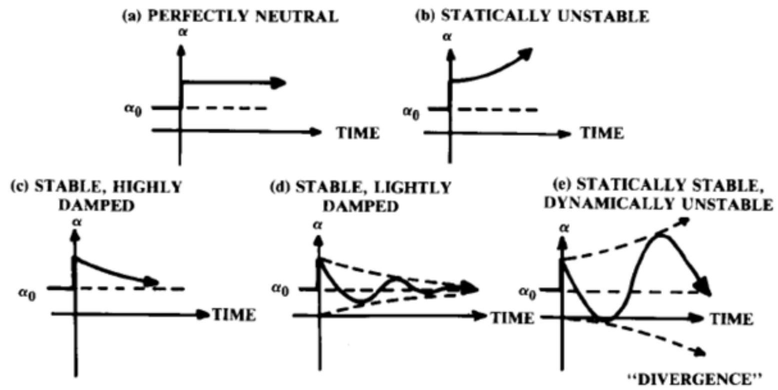
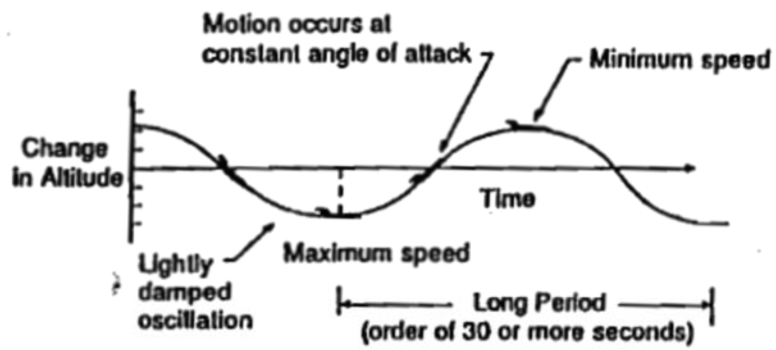
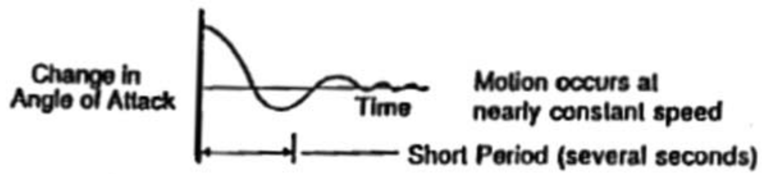
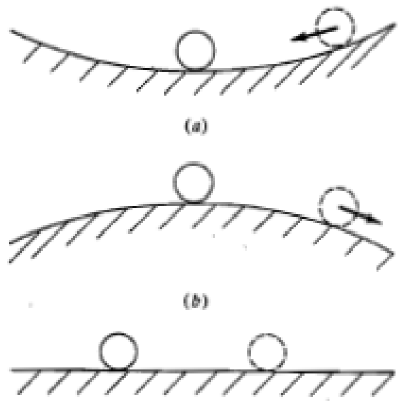


Fig. 16.1 Static and dynamic stability.





Estabilidad Estática y Dinámica



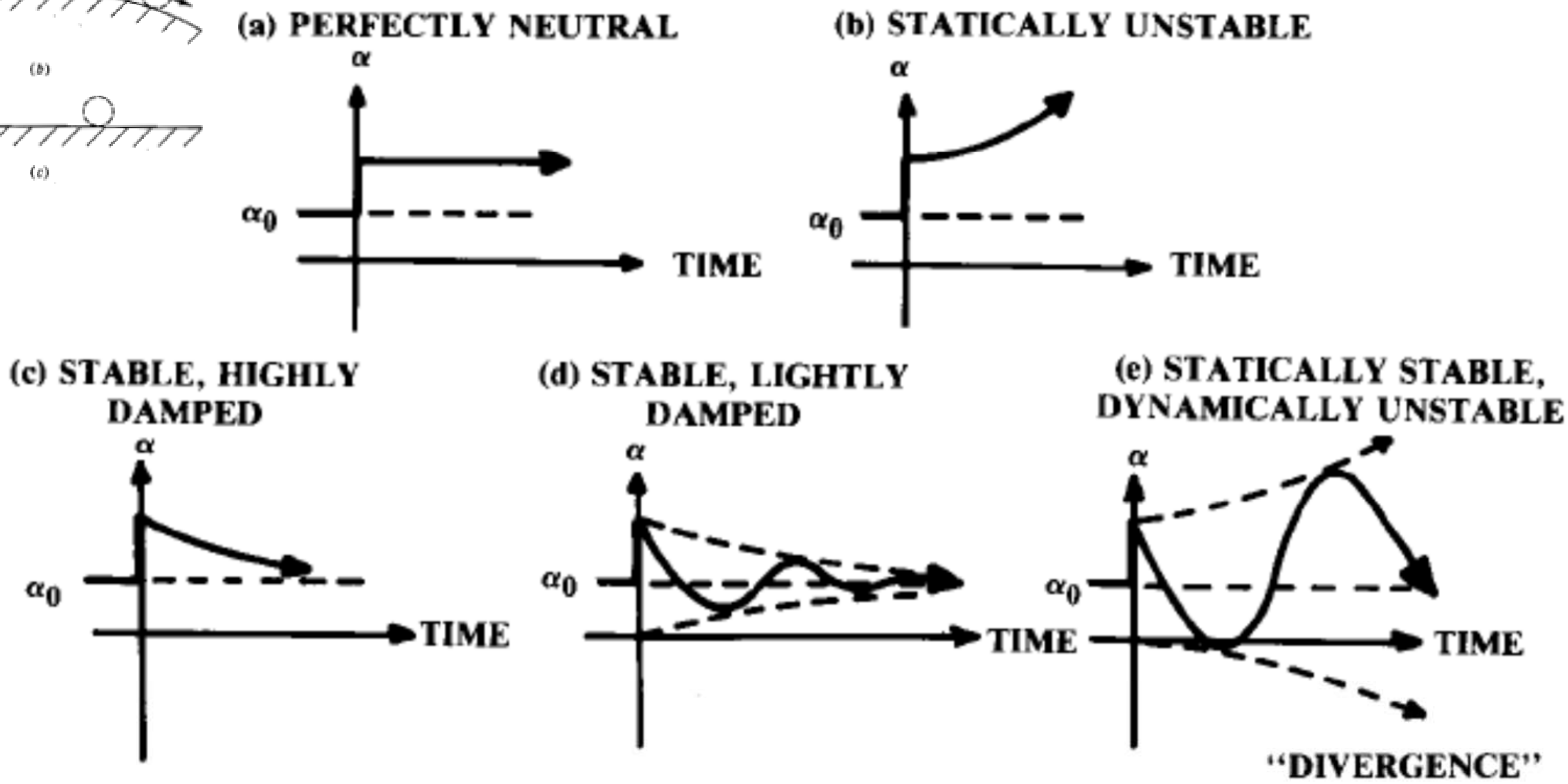
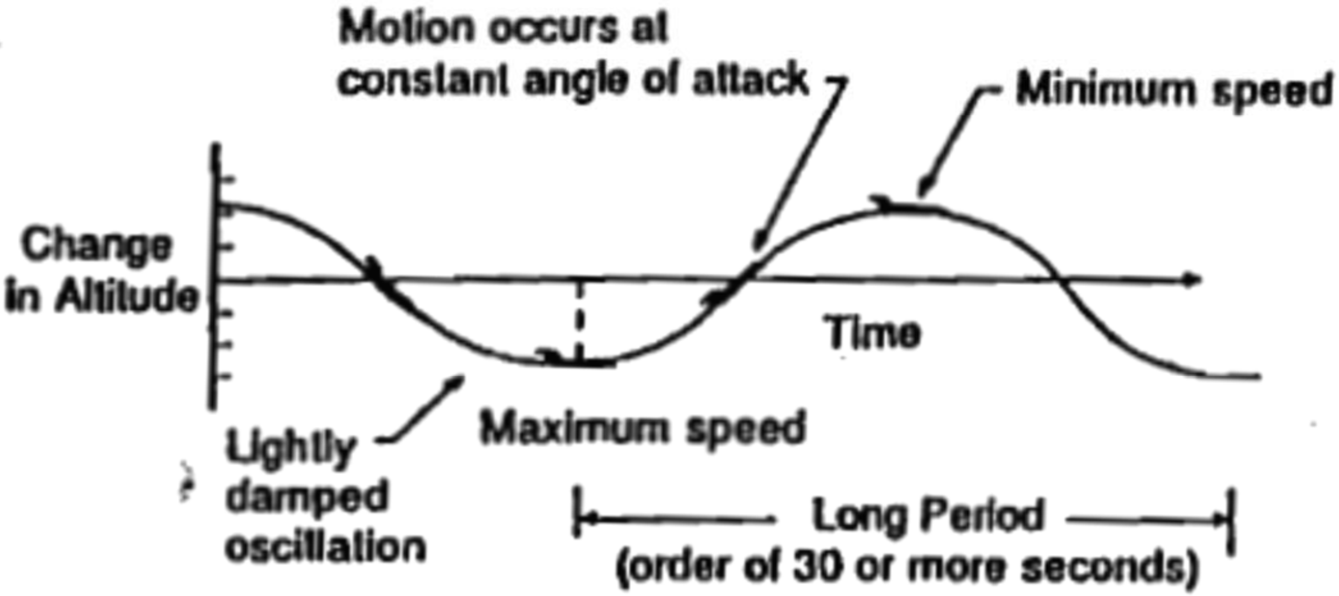
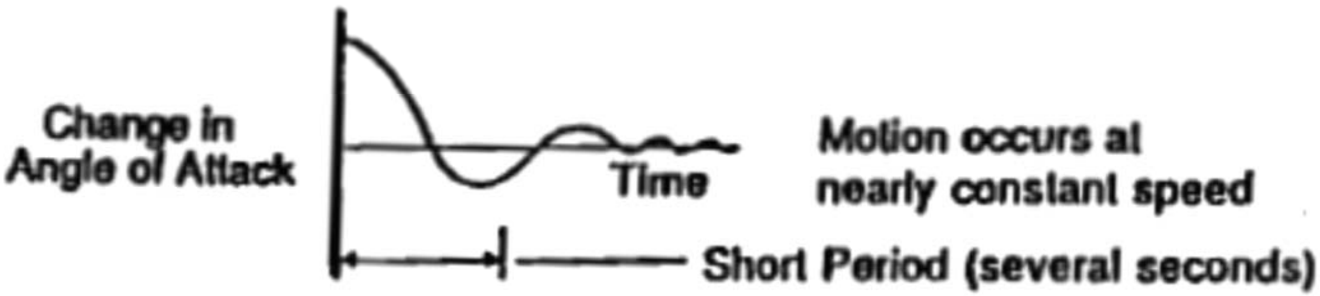


Fig. 16.1 Static and dynamic stability.



Motion occurs at
constant angle of attack
Minimum speed
Change
in Altitude
Time
Lightly
damped
oscillation
Maximum speed
Long Period
(order of 30 or more seconds)



Change in
Angle of Attack
Time
Motion occurs at
nearly constant speed
Short Period (several seconds)

Análisis de Estabilidad Longitudinal - I

- **Uso de la teoria de pequeñas perturbaciones para obtener las matrices invarienteas con el tiempo - Linear Time Invariant Matrix (LTI):**
  - **u - forward speed**
  - **α - angle of attack (AoA)**
  - **q - pitch rate**
  - **θ - pitch angle**
  - **δ_e – elevator deflection**

$$\begin{bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} (X_u + X_{T_u}) & X_\alpha & X_q & -g\cos\theta_1 \\ \frac{Z_u}{U_1 - Z_{\dot{\alpha}}} & \frac{Z_\alpha}{U_1 - Z_{\dot{\alpha}}} & \frac{(Z_q + U_1)}{U_1 - Z_{\dot{\alpha}}} & \frac{-g\sin\theta_1}{U_1 - Z_{\dot{\alpha}}} \\ \frac{M_{\dot{\alpha}} Z_u}{U_1 - Z_{\dot{\alpha}}} + (M_u + M_{T_u}) & \frac{M_{\dot{\alpha}} Z_\alpha}{U_1 - Z_{\dot{\alpha}}} + (M_\alpha + M_{T_\alpha}) & \frac{M_{\dot{\alpha}}(Z_q + U_1)}{U_1 - Z_{\dot{\alpha}}} + M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ \frac{Z_{\delta_e}}{U_1 - Z_{\dot{\alpha}}} \\ \frac{M_{\dot{\alpha}} Z_{\delta_e}}{U_1 - Z_{\dot{\alpha}}} + M_{\delta_e} \\ 0 \end{bmatrix} \delta_e$$

Análisis de Estabilidad Longitudinal - II

For the longitudinal equations:

$$m\dot{u} = -mg\cos\theta_1 + \bar{q}_1 S\left\{-(C_{D_u} + 2C_{D_1})\frac{u}{U_1} + (C_{T_{x_u}} + 2C_{T_{x_1}})\frac{u}{U_1}\right\} +$$

$$+\bar{q}_1 S\left\{\frac{C_D}{U_1} - (C_{D_\alpha} - C_{L_1})\alpha - C_{D_{\delta_e}}\delta_e\right\}$$

$$m(\dot{w} - U_1 q) = -mg\sin\theta_1 + \bar{q}_1 S\left\{-(C_{L_u} + 2C_{L_1})\frac{u}{U_1} - (C_{L_\alpha} + C_{D_1})\alpha\right\} +$$

$$+\bar{q}_1 S\left\{-C_{L_{\dot{\alpha}}}\frac{\alpha\bar{c}}{2U_1} - C_{L_q}\frac{q\bar{c}}{2U_1} - C_{L_{\delta_e}}\delta_e\right\}$$

$$I_{yy}\dot{q} = \bar{q}_1 S\bar{c}\left\{(C_{m_u} + 2C_{m_1})\frac{u}{U_1} + (C_{m_{T_u}} + 2C_{m_{T_1}})\frac{u}{U_1} + C_{m_\alpha}\alpha + C_{m_{T_\alpha}}\alpha\right\} +$$

$$+\bar{q}_1 S\bar{c}\left\{C_{m_{\dot{\alpha}}}\frac{\alpha\bar{c}}{2U_1} + C_{m_q}\frac{q\bar{c}}{2U_1} + C_{m_{\delta_e}}\delta_e\right\}$$

$$\text{where: } \quad q = \dot{\theta} \quad \text{and} \quad w = U_1\alpha$$

Análisis de Estabilidad Longitudinal - III

perturbed longitudinal equations with dimensional stability derivatives

$$\dot{u} = -g\theta\cos\theta_1 + X_u u + X_{T_u} u + X_\alpha \alpha + X_{\delta_e}\delta_e$$

$$U_1\dot{\alpha} - U_1\dot{\theta} = -g\theta\sin\theta_1 + Z_u u + Z_\alpha \alpha + Z_{\dot{\alpha}}\dot{\alpha} + Z_q\dot{\theta} + Z_{\delta_e}\delta_e$$

$$\ddot{\theta} = M_u u + M_{T_u} u + M_\alpha \alpha + M_{T_\alpha}\alpha + M_{\dot{\alpha}}\dot{\alpha} + M_q\dot{\theta} + M_{\delta_e}\delta_e$$

$$q = \dot{\theta}$$

: state-space matrix model for the longitudinal mode

$$\dot{X}_{lon} = E_{lon}^{-1} A_{lon} X_{lon} + E_{lon}^{-1} B_{lon} U_{lon}$$

state and the control vectors

$$X_{lon} = \begin{bmatrix} u & \alpha & q & \theta \end{bmatrix}^T$$

$$U_{lon} = \begin{bmatrix} \delta_e \end{bmatrix}$$

Análisis de Estabilidad Longitudinal - IV

Modelo Matricial de “State Space”

$$\begin{bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} (X_u + X_{T_u}) & X_\alpha & X_q & -g\cos\theta_1 \\ \frac{Z_u}{U_1 - Z_{\dot{\alpha}}} & \frac{Z_\alpha}{U_1 - Z_{\dot{\alpha}}} & \frac{(Z_q + U_1)}{U_1 - Z_{\dot{\alpha}}} & \frac{-g\sin\theta_1}{U_1 - Z_{\dot{\alpha}}} \\ \frac{M_{\dot{\alpha}} Z_u}{U_1 - Z_{\dot{\alpha}}} + (M_u + M_{T_u}) & \frac{M_{\dot{\alpha}} Z_\alpha}{U_1 - Z_{\dot{\alpha}}} + (M_\alpha + M_{T_\alpha}) & \frac{M_{\dot{\alpha}}(Z_q + U_1)}{U_1 - Z_{\dot{\alpha}}} + M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ q \\ \theta \end{bmatrix} +$$

$$\begin{bmatrix} X_{\delta_e} \\ \frac{Z_{\delta_e}}{U_1 - Z_{\dot{\alpha}}} \\ \frac{M_{\dot{\alpha}} Z_{\delta_e}}{U_1 - Z_{\dot{\alpha}}} + M_{\delta_e} \\ 0 \end{bmatrix} \delta_e$$

Análisis de Estabilidad Longitudinal - V

Derivadas de estabilidad dimensionales

$$X_u = \frac{-\bar{q}_1 S (C_{D_u} + 2C_{D_1})}{mU_1}$$

$$X_{T_u} = \frac{\bar{q}_1 S (C_{T_{x_u}} + 2C_{T_{x_1}})}{mU_1}$$

$$X_\alpha = \frac{-\bar{q}_1 S (C_{D_\alpha} - C_{L_1})}{m}$$

$$X_{\delta_e} = \frac{-\bar{q}_1 S C_{D_{\delta_e}}}{m}$$

$$Z_u = \frac{-\bar{q}_1 S (C_{L_u} + 2C_{L_1})}{mU_1}$$

$$Z_\alpha = \frac{-\bar{q}_1 S (C_{L_\alpha} + C_{D_1})}{m}$$

$$Z_{\dot{\alpha}} = \frac{-\bar{q}_1 S \bar{c} C_{L_{\dot{\alpha}}}}{2mU_1}$$

$$Z_q = \frac{-\bar{q}_1 S \bar{c} C_{L_q}}{2mU_1}$$

$$Z_{\delta_e} = \frac{-\bar{q}_1 S C_{L_{\delta_e}}}{m}$$

$$M_u = \frac{\bar{q}_1 S \bar{c} (C_{m_u} + 2C_{m_1})}{I_{yy} U_1}$$

$$M_{T_u} = \frac{\bar{q}_1 S \bar{c} (C_{m_{T_u}} + 2C_{m_{T_1}})}{I_{yy} U_1}$$

$$M_\alpha = \frac{\bar{q}_1 S \bar{c} C_{m_\alpha}}{I_{yy}}$$

$$M_{T_\alpha} = \frac{\bar{q}_1 S \bar{c} C_{m_{T_\alpha}}}{I_{yy}}$$

$$M_{\dot{\alpha}} = \frac{\bar{q}_1 S \bar{c}^2 C_{m_{\dot{\alpha}}}}{2 I_{yy} U_1}$$

$$M_q = \frac{\bar{q}_1 S \bar{c}^2 C_{m_q}}{2 I_{yy} U_1}$$

$$M_{\delta_e} = \frac{\bar{q}_1 S \bar{c} C_{m_{\delta_e}}}{I_{yy}}$$

Formulación Pamadi - I

TablaDerivadas

Longitudinal

Cx_alpha	0.2339
Cz_alpha	-5.0523
Cm_alpha	-0.1736
Cx_q	0
Cz_q	-3.6216
Cm_q	-8.2617
Cx_u	-0.0710
Cz_u	-0.8095
Cm_u	0
Cx_alpha_dot	0
Cz_alpha_dot	61.8215
Cm_alpha_dot	5.7756

Lateral

Cy_beta	-0.2738
Cl_beta	-0.0094
Cn_beta	0.0543
Cy_p	0.0272
Cl_p	-0.4061
Cn_p	-0.1196
Cy_r	0.3396
Cl_r	0.1530
Cn_r	-0.1711
Cy_beta_dot	0.0637
Cl_beta_dot	0.0080
Cn_beta_dot	-0.0289

Control Longitudinal

Cl_delta_e	1.1440
Cm_delta_e	-3.6608

Control Lateral

Cy_delta_a	0
Cl_delta_a	0.1541
Cn_delta_a	0.1225
Cy_delta_r	0.1334
Cl_delta_r	0.0121
Cn_delta_r	-0.0537

Trim Longitudinal

Trim Lateral

Dinámica

Exportar Datos

Volver

Pablo García Mascort, Universidad de Sevilla, 2014

Formulación Pamadi - II

$$\frac{du}{dt}=\frac{1}{m_1}[(C_{xu}+\xi_1 C_{zu})u+(C_{x\alpha}+\xi_1 C_{z\alpha})\Delta\alpha$$
$$+\left[C_{xq}c_1+\xi_1(m_1+C_{zq}c_1)\right]q+(C_{x\theta}+\xi_1 C_{z\theta})\Delta\theta+(C_{x\delta_e}+\xi_1 C_{z\delta_e})\Delta\delta_e]$$

$$\dot{X}=AX+BU$$

$$\xi_1=\frac{C_{x\dot{\alpha}}c_1}{m_1-C_{z\dot{\alpha}}c_1}$$

$$\frac{d\Delta\alpha}{dt}=\frac{1}{(m_1-C_{z\dot{\alpha}}c_1)}[C_{zu}u+C_{z\alpha}\Delta\alpha+(m_1+C_{zq}c_1)q+C_{z\theta}\Delta\theta+C_{z\delta_e}\Delta\delta_e]$$

$$\xi_2=\frac{C_{m\dot{\alpha}}c_1}{m_1-C_{z\dot{\alpha}}c_1}$$

$$\frac{dq}{dt}=\frac{1}{I_{y1}}[(C_{mu}+\xi_2 C_{zu})u+(C_{m\alpha}+\xi_2 C_{z\alpha})\Delta\alpha$$
$$+\left[C_{mq}c_1+\xi_2(m_1+C_{zq}c_1)\right]q+\xi_2 C_{z\theta}\Delta\theta+(C_{m\delta_e}+\xi_2 C_{z\delta_e})\Delta\delta_e]$$

$$m_1=\frac{2m}{\rho U_o S}\qquad c_1=\frac{\bar{c}}{2U_o}\qquad I_{y1}=\frac{I_y}{\frac{1}{2}\rho U_o^2 S\bar{c}}$$

$$\frac{d\Delta\theta}{dt}=q$$

$$x_1=u\qquad x_2=\Delta\alpha\qquad x_3=q\qquad x_4=\Delta\theta$$

$$X=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\qquad A=\begin{bmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\a_{21}&a_{22}&a_{23}&a_{24}\\a_{31}&a_{32}&a_{33}&a_{34}\\a_{41}&a_{42}&a_{43}&a_{44}\end{bmatrix}\qquad B=\begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}\qquad U=\delta_e$$

Formulación Pamadi - III

$$a_{11} = \frac{C_{xu} + \xi_1 C_{zu}}{m_1} \qquad a_{12} = \frac{C_{x\alpha} + \xi_1 C_{z\alpha}}{m_1} \qquad a_{13} = \frac{C_{xq} c_1 + \xi_1 (m_1 + C_{zq} c_1)}{m_1}$$

$$a_{14} = \frac{C_{x\theta} + \xi_1 C_{z\theta}}{m_1} \qquad a_{21} = \frac{C_{zu}}{m_1 - C_{z\dot{\alpha}} c_1} \qquad a_{22} = \frac{C_{z\alpha}}{m_1 - C_{z\dot{\alpha}} c_1} \qquad a_{23} = \frac{m_1 + C_{zq} c_1}{m_1 - C_{z\dot{\alpha}} c_1}$$

$$a_{24} = \frac{C_{z\theta}}{m_1 - C_{z\dot{\alpha}} c_1} \qquad a_{31} = \frac{C_{mu} + \xi_2 C_{zu}}{I_{y1}} \qquad a_{32} = \frac{C_{m\alpha} + \xi_2 C_{z\alpha}}{I_{y1}}$$

$$a_{33} = \frac{C_{mq} c_1 + \xi_2 (m_1 + C_{zq} c_1)}{I_{y1}} \qquad a_{34} = \frac{\xi_2 C_{z\theta}}{I_{y1}}$$

$$a_{41} = 0 \qquad a_{42} = 0 \qquad a_{43} = 1 \qquad a_{44} = 0$$

$$b_1 = \frac{C_{x\delta_e} + \xi_1 C_{z\delta_e}}{m_1} \qquad b_2 = \frac{C_{z\delta_e}}{m_1 + c_1 C_{z\dot{\alpha}}} \qquad b_3 = \frac{C_{m\delta_e} + \xi_2 C_{z\delta_e}}{I_{y1}} \qquad b_4 = 0$$

Conversión nomenclatura

$$Cx_{\alpha} = C_L - C_{D_{\alpha}}$$
$$C_{Z_{\alpha}} = -C_{L_{\alpha}} - C_D$$

$$C_D = C_{D_0} + C_{D_1} C_L + C_{D_2} C_L^2$$
$$C_{D\alpha} = (C_{D1} + 2C_{D2} C_L) C_{L_{\alpha}}$$

$$C_{X_u} = -3C_D - C_{D_u}$$

$$C_{T_x} = C_D$$
$$C_{T\,x_u} = -3C_{T_x}$$

$$C_{M\,T_u} = -\left(\frac{d_T}{c}\right) C_{T\,x_u}$$
$$C_{M_{T_{\alpha}}} \approx 0$$
$$C_{M_{T_{\alpha}}} \approx 0$$

$$C_{Z_u} = -2C_L - C_{L_u}$$

$$C_{L_u} \approx 0$$

$$C_{X_q} = -C_{D_q}$$
$$C_{Z_q} = -C_{L_q}$$
$$C_{X_{\dot{\alpha}}} = -C_{D_{\dot{\alpha}}}$$
$$C_{Z_{\dot{\alpha}}} = -C_{L_{\dot{\alpha}}}$$

$$C_{D_q} \approx 0$$

Análisis de Estabilidad Lateral-Direccional - I

- **Uso de la teoria de pequeñas perturbaciones para obtener las matrices invarienteas con el tiempo - Linear Time Invariant Matrix (LTI):**
  - **v – side-slip velocity**
  - **p – roll rate**
  - **r - yaw rate**
  - **ϕ - bank angle**
  - **ψ - heading angle**
  - **δ_a – aileron deflection**

$$\overline{A}_1 = \frac{I_{xz}}{I_{xx}} \quad \overline{B}_1 = \frac{I_{xz}}{I_{zz}}$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{U_1} & Y_p & Y_r - U_1 & g\cos\theta_0 & 0 \\ \frac{L_\beta + A_1[N_\beta + N_{T\beta}]}{(1-A_1B_1)U_1} & \frac{L_p + A_1N_p}{1-A_1B_1} & \frac{L_r + A_1N_r}{1-A_1B_1} & 0 & 0 \\ \frac{B_1L_\beta + N_\beta + N_{T\beta}}{(1-A_1B_1)U_1} & \frac{B_1L_p + N_p}{1-A_1B_1} & \frac{B_1L_r + N_r}{1-A_1B_1} & 0 & 0 \\ 0 & 1 & \tan\theta_0 & 0 & 0 \\ 0 & 0 & \frac{1}{\cos\theta_0} & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \\ \psi \end{bmatrix} + \begin{bmatrix} Y_{\delta_A} & Y_{\delta_r} \\ \frac{L_{\delta_A} + A_1N_{\delta_A}}{1-A_1B_1} & \frac{L_{\delta_r} + A_1N_{\delta_r}}{1-A_1B_1} \\ \frac{B_1L_{\delta_A} + N_{\delta_A}}{1-A_1B_1} & \frac{B_1L_{\delta_r} + N_{\delta_r}}{1-A_1B_1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

For the lateral-directional equations:

$$m(\dot{v} + U_1 r) = mg\phi\cos\theta_1 + \bar{q}_1 S\left\{C_{y_\beta}\beta + C_{y_p}\frac{pb}{2U_1} + C_{y_r}\frac{rb}{2U_1} + C_{y_{\delta_a}}\delta_a + C_{y_{\delta_r}}\delta_r\right\}$$

$$I_{xx}\dot{p} - I_{xz}\dot{r} = \bar{q}_1 Sb\left\{C_{l_\beta}\beta + C_{l_p}\frac{pb}{2U_1} + C_{l_r}\frac{rb}{2U_1} + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r\right\}$$

$$I_{zz}\dot{r} - I_{xz}\dot{p} = \bar{q}_1 Sb\left\{C_{n_\beta}\beta + C_{n_{T_\beta}}\beta + C_{n_p}\frac{pb}{2U_1} + C_{n_r}\frac{rb}{2U_1} + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r\right\}$$

where : $p = \dot{\phi}$, $r = \dot{\psi}$ and $v = U_1\beta$

Análisis de Estabilidad Lateral-Direccional - III

The perturbed lateral/directional equations with dimensional stability derivatives become:

$$U_1\dot{\beta} + U_1\dot{\psi} = g\phi\cos\theta_1 + Y_\beta\beta + Y_p\dot{\phi} + Y_r\dot{\psi} + Y_{\delta_a}\delta_a + Y_{\delta_r}\delta_r$$

$$\ddot{\phi} - \overline{A}_1\ddot{\psi} = L_\beta\beta + L_p\dot{\phi} + L_r\dot{\psi} + L_{\delta_a}\delta_a + L_{\delta_r}\delta_r \qquad \overline{A}_1 = \frac{I_{xz}}{I_{xx}}$$

$$\ddot{\psi} - \overline{B}_1\ddot{\phi} = N_\beta\beta + N_{T_\beta}\beta + N_p\dot{\phi} + N_r\dot{\psi} + N_{\delta_a}\delta_a + N_{\delta_r}\delta_r \qquad \overline{B}_1 = \frac{I_{xz}}{I_{zz}}$$

$$p = \dot{\phi} - \dot{\psi}\sin\Theta_1$$

$$r = \dot{\psi}\cos\Theta_1$$

state-space matrix model for the lateral/directional mode

$$\dot{X}_{lat} = E_{lat}^{-1}A_{lat}X_{lat} + E_{lat}^{-1}B_{lat}U_{lat}$$

state and the control vectors

$$X_{lat} = \begin{bmatrix} \beta & p & r & \phi & \psi \end{bmatrix}^T$$

$$U_{lat} = \begin{bmatrix} \delta_a & \delta_r \end{bmatrix}^T$$

Análisis de Estabilidad Lateral-Direccional - IV

Modelo Matricial de "State Space"

$$
\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} =
\begin{bmatrix}
\frac{Y_\beta}{U_1} & Y_p & Y_r - U_1 & g\cos\theta_0 & 0 \\
\frac{L_\beta + A_1[N_\beta + N_{T\beta}]}{(1-A_1B_1)U_1} & \frac{L_p + A_1 N_p}{1-A_1B_1} & \frac{L_r + A_1 N_r}{1-A_1B_1} & 0 & 0 \\
\frac{B_1 L_\beta + N_\beta + N_{T\beta}}{(1-A_1B_1)U_1} & \frac{B_1 L_p + N_p}{1-A_1B_1} & \frac{B_1 L_r + N_r}{1-A_1B_1} & 0 & 0 \\
0 & 1 & \tan\theta_0 & 0 & 0 \\
0 & 0 & \frac{1}{\cos\theta_0} & 0 & 0
\end{bmatrix}
\begin{bmatrix} \beta \\ p \\ r \\ \phi \\ \psi \end{bmatrix} +
$$

$$
\begin{bmatrix}
Y_{\delta_A} & Y_{\delta_r} \\
\frac{L_{\delta_A} + A_1 N_{\delta_A}}{1-A_1B_1} & \frac{L_{\delta_r} + A_1 N_{\delta_r}}{1-A_1B_1} \\
\frac{B_1 L_{\delta_A} + N_{\delta_A}}{1-A_1B_1} & \frac{B_1 L_{\delta_r} + N_{\delta_r}}{1-A_1B_1} \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}
$$

Análisis de Estabilidad Lateral-Direccional - V

Derivadas de estabilidad dimensionales

$$Y_\beta = \frac{\bar{q}_1 S C_{y_\beta}}{m} \qquad L_\beta = \frac{\bar{q}_1 S b C_{l_\beta}}{I_{xx}}$$

$$Y_p = \frac{\bar{q}_1 S b C_{y_p}}{2mU_1} \qquad L_p = \frac{\bar{q}_1 S b^2 C_{l_p}}{2I_{xx}U_1}$$

$$Y_r = \frac{\bar{q}_1 S b C_{y_r}}{2mU_1} \qquad L_r = \frac{\bar{q}_1 S b^2 C_{l_r}}{2I_{xx}U_1}$$

$$Y_{\delta_a} = \frac{\bar{q}_1 S C_{y_{\delta_a}}}{m} \qquad L_{\delta_a} = \frac{\bar{q}_1 S b C_{l_{\delta_a}}}{I_{xx}}$$

$$Y_{\delta_r} = \frac{\bar{q}_1 S C_{y_{\delta_r}}}{m} \qquad L_{\delta_r} = \frac{\bar{q}_1 S b C_{l_{\delta_r}}}{I_{xx}}$$

$$N_\beta = \frac{\bar{q}_1 S b C_{n_\beta}}{I_{zz}} \qquad N_r = \frac{\bar{q}_1 S b^2 C_{n_r}}{2I_{zz}U_1}$$

$$N_{T_\beta} = \frac{\bar{q}_1 S b C_{n_{T_\beta}}}{I_{zz}} \qquad N_{\delta_a} = \frac{\bar{q}_1 S b C_{n_{\delta_a}}}{I_{zz}}$$

$$N_p = \frac{\bar{q}_1 S b^2 C_{n_p}}{2I_{zz}U_1} \qquad N_{\delta_r} = \frac{\bar{q}_1 S b C_{n_{\delta_r}}}{I_{zz}}$$

$$C_{N_{T\beta}} \approx 0$$

Formulación Pamadi - I

$$\frac{d\beta}{dt}=\left(\frac{1}{m_1-b_1C_{y\dot\beta}}\right)[C_{y\beta}\Delta\beta+C_{y\phi}\Delta\phi+b_1C_{yp}p-(m_1-b_1C_{yr})r+C_{y\delta_a}\Delta\delta_a+C_{y\delta_r}\Delta\delta_r]$$

$$\dot p=\frac{1}{I_{x1}}[C_{l\beta}\Delta\beta+C_{l\dot\beta}b_1\Delta\dot\beta+b_1C_{lp}p+b_1C_{lr}r+I_{xz1}\dot r+C_{l\delta_a}\Delta\delta_a+C_{l\delta_r}\Delta\delta_r]$$

$$\dot r=\frac{1}{I_{z1}}[C_{n\beta}\Delta\beta+C_{n\dot\beta}b_1\Delta\dot\beta+b_1C_{np}p+b_1C_{nr}r+I_{xz1}\dot p+C_{n\delta_a}\Delta\delta_a+C_{n\delta_r}\Delta\delta_r]$$

$$I_{x1}=\frac{I_x}{\frac{1}{2}\rho U_o^2Sb}\qquad I_{z1}=\frac{I_z}{\frac{1}{2}\rho U_o^2Sb}\qquad I_{xz1}=\frac{I_{xz}}{\frac{1}{2}\rho U_o^2Sb}$$

$$I'_{x1}=\frac{I_{x1}}{I_{x1}I_{z1}-I_{xz1}^2}\qquad I'_{z1}=\frac{I_{z1}}{I_{x1}I_{z1}-I_{xz1}^2}\qquad I'_{xz1}=\frac{I_{xz1}}{I_{x1}I_{z1}-I_{xz1}^2}$$

$$b_1=\frac{b}{2U_o}$$

$$x=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=\begin{bmatrix}\Delta\beta\\\Delta\phi\\p\\\Delta\psi\\r\end{bmatrix}\qquad A=\begin{bmatrix}a_{11}&a_{12}&a_{13}&a_{14}&a_{15}\\a_{21}&a_{22}&a_{23}&a_{24}&a_{25}\\a_{31}&a_{32}&a_{33}&a_{34}&a_{35}\\a_{41}&a_{42}&a_{43}&a_{44}&a_{45}\\a_{51}&a_{52}&a_{53}&a_{54}&a_{55}\end{bmatrix}\qquad B=\begin{bmatrix}b_{11}&b_{12}\\b_{21}&b_{22}\\b_{31}&b_{32}\\b_{41}&b_{42}\end{bmatrix}\qquad U=\begin{bmatrix}\Delta\delta_a\\\Delta\delta_r\end{bmatrix}$$

Formulación Pamadi - II

$$a_{11} = \frac{C_{y\beta}}{m_1 - b_1 C_{y\dot{\beta}}} \qquad a_{12} = \frac{C_{y\phi}}{m_1 - b_1 C_{y\dot{\beta}}} \qquad a_{13} = \frac{C_{yp} b_1}{m_1 - b_1 C_{y\dot{\beta}}}$$

$$a_{14} = 0 \qquad a_{15} = -\left(\frac{m_1 - b_1 C_{yr}}{m_1 - b_1 C_{y\dot{\beta}}} \right)$$

$$a_{21} = 0 \qquad a_{22} = 0 \qquad a_{23} = 1 \qquad a_{24} = 0 \qquad a_{25} = 0$$

$$a_{31} = C_{l\beta} I'_{z1} + C_{n\beta} I'_{xz1} + \xi_1 b_1 a_{11} \qquad a_{32} = \xi_1 b_1 a_{12}$$

$$a_{33} = C_{lp} b_1 I'_{z1} + C_{np} I'_{xz1} b_1 + \xi_1 b_1 a_{13} \qquad a_{34} = 0$$

$$a_{35} = C_{lr} b_1 I'_{z1} + C_{nr} I'_{xz1} b_1 + \xi_1 b_1 a_{15}$$

$$a_{41} = a_{42} = a_{43} = a_{44} = 0 \qquad a_{45} = 1 \qquad a_{51} = C_{n\beta} I'_{x1} + C_{l\beta} I'_{xz1} + \xi_2 b_1 a_{11}$$

$$a_{52} = \xi_2 b_1 a_2 \qquad a_{53} = b_1 (C_{np} I'_{x1} + C_{lp} I'_{xz1} + \xi_2 a_{11})$$

$$a_{54} = 0 \qquad a_{55} = b_1 (C_{nr} I'_{x1} + C_{lr} I'_{xz1} + \xi_2 a_{15})$$

Formulación Pamadi - III

$$b_{11} = \frac{C_{y\delta_a}}{(m_1 - b_1 C_{y\beta})} \qquad b_{12} = \frac{C_{y\delta_r}}{(m_1 - b_1 C_{y\beta})}$$

$$b_{21} = 0 \qquad b_{22} = 0$$

$$b_{31} = C_{l\delta_a} I'_{z1} + C_{n\delta_a} I'_{xz1} + \xi_1 b_1 b_{11} \qquad b_{32} = C_{l\delta_r} I'_{z1} + C_{n\delta_r} I'_{xz1} + \xi_1 b_1 b_{12}$$

$$b_{41} = 0 \qquad b_{42} = 0$$

$$b_{51} = C_{n\delta_a} I'_{x1} + C_{l\delta_a} I'_{xz1} + \xi_2 b_1 b_{11} \qquad b_{52} = C_{n\delta_r} I'_{x1} + C_{l\delta_r} I'_{xz1} + \xi_2 b_1 b_{12}$$

$$\xi_1 = I'_{z1} C_{l\dot{\beta}} + I'_{xz1} C_{n\dot{\beta}} \qquad \xi_2 = I'_{x1} C_{n\dot{\beta}} + I'_{xz1} C_{l\dot{\beta}}$$

Criterios de Estabilidad Estática

- **Longitudinal:**
  - $C_{M,0}$ **must be zero.**
  - $\partial C_{M,cg} / \partial \alpha_a$ **must be negative.**
- **Lateral:**
  - C_{l_β} **must be negative with magnitude half of** C_{n_β}
- **Dynamic lateral stability criteria :**
  - **Class airplane I Flight Phase regime A:**
    - **Minimum Dutch damping ratio of 0.19**
    - **Minimum Dutch natural frequency 1.0 rad/sec**

Table 4.1 Criteria for Static Stability of Airplanes

Forces and moments	Perturbed Variables: u	v	w	$\beta = \frac{v}{U_1}$	$\alpha = \frac{w}{U_1}$	p	q	r
$F_{A_x} + F_{T_x}$	$\frac{\partial(F_{A_x} + F_{T_x})}{\partial u} < 0$ $\approx C_{D_u} > 0$							
$F_{A_y} + F_{T_y}$		$\frac{\partial(F_{A_y} + F_{T_y})}{\partial v} < 0$ $\approx C_{y_\beta} < 0$						
$F_{A_z} + F_{T_z}$			$\frac{\partial(F_{A_z} + F_{T_z})}{\partial w} < 0$ $\approx C_{L_\alpha} > 0$					
$L_A + L_T$				$\frac{\partial(L_A + L_T)}{\partial \beta} < 0$ $\approx C_{l_\beta} < 0$		$\frac{\partial(L_A + L_T)}{\partial p} < 0$ $\approx C_{l_p} < 0$		
$M_A + M_T$	$\frac{\partial(M_A + M_T)}{\partial u} > 0$ $\approx C_{m_u} > 0$				$\frac{\partial(M_A + M_T)}{\partial \alpha} > 0$ $\approx C_{m_\alpha} < 0$		$\frac{\partial(M_A + M_T)}{\partial q} < 0$ $\approx C_{m_q} < 0$	
$N_A + N_T$				$\frac{\partial(N_A + N_T)}{\partial \beta} > 0$ $\approx C_{n_\beta} > 0$				$\frac{\partial(N_A + N_T)}{\partial r} < 0$ $\approx C_{n_r} < 0$

Notes: 1. All perturbations are taken relative to a steady state: $U_1, V_1, W_1, P_1, Q_1, R_1$
2. Blanks in the table indicate that there is no stability consequence

Autovalores

Autovalores

$$\lambda = n \mp \omega$$

$n \rightarrow$ parte real del autovalor

$\omega \rightarrow$ parte imaginaria del autovalor

$\omega_n = \sqrt{n^2 + \omega^2} \rightarrow$ frecuencia natural

$\zeta = -\frac{n}{\omega_n} \rightarrow$ amortiguamiento

$$\omega = \omega_n \sqrt{1 - \zeta^2}$$

$$n = -\zeta \omega_n$$

Time to double or half

$$t_{double} \; or \; t_{half} = \frac{0.693}{|n|} = \frac{0.693}{|\zeta|\omega_n}$$

Cycles to double or half

$$N_{double} \; or \; N_{half} = 0.110 \frac{\omega}{|n|} = 0.110 \frac{\sqrt{1 - \zeta^2}}{|\zeta|}$$

Logarithmic decrement

$$\delta = \log_e \frac{e^{nt}}{e^{n(t+T)}} = -nT = 2\pi \frac{\zeta}{\sqrt{1-\zeta^2}} = -\frac{0.693}{N_{double}} = \frac{0.693}{N_{half}}$$

Aproximaciones - Longitudinal

short period undamped natural frequency and damping ratio:

$$\omega_{n_{sp}} \approx \sqrt{\frac{Z_\alpha M_q}{U_1} - M_\alpha} \qquad \zeta_{sp} \approx \frac{-\left(M_q + \frac{Z_{\dot\alpha}}{U_1} + M_{\dot\alpha}\right)}{2\omega_{n_{sp}}}$$

autovalores

$$s_{sp} = -\zeta_{sp}\omega_{n_{sp}} \pm j\omega_{n_{sp}}\sqrt{1 - \zeta_{sp}^{2}}$$

phugoid undamped natural frequency and damping ratio:

$$\omega_{n_{ph}} \approx \sqrt{\frac{-gZ_u}{U_1}} \qquad \zeta_{ph} \approx \frac{-X_u}{2\omega_{n_{ph}}}$$

autovalores

$$s_{ph} = -\zeta_{ph}\omega_{n_{ph}} \pm j\omega_{n_{ph}}\sqrt{1 - \zeta_{ph}^{2}}$$

Aproximaciones – Lateral-Direccional

dutch roll undamped natural frequency and damping ratio:

$$\omega_{n_d} \approx \sqrt{\left\{N_\beta + \frac{1}{U_1}(Y_\beta N_r - N_\beta Y_r)\right\}}$$

$$\zeta_d \approx \frac{-\left(N_r + \frac{Y_\beta}{U_1}\right)}{2\omega_{n_d}}$$

approximate spiral root

$$s_3 = s_{spiral} = \frac{(L_\beta N_r - N_\beta L_r)}{(L_\beta + N_\beta \bar{A}_1)}$$

$$T_s \approx - s_{spiral}$$

the criterion for spiral root stability

$$(L_\beta N_r - N_\beta L_r) > 0$$

$$\bar{A}_1 = \frac{I_{xz}}{I_{xx}} \quad \bar{B}_1 = \frac{I_{xz}}{I_{zz}}$$

rolling approximation

$$s_4 = s_{roll} \approx L_p \qquad T_r \approx -1/L_p$$

Criterios Estabilidad Estática - I

A static stability criterion is defined as a rule by which steady state flight conditions are separated into the categories of stable, unstable or neutrally stable..

FORWARD SPEED STABILITY

$$\frac{\partial(F_{A_x} + F_{T_x})}{\partial u} < 0 \qquad F_{A_x} + F_{T_x} = (-C_D + C_{T_x})\bar{q}S$$

$$(C_{T_{x_u}} - C_{D_u}) + (C_{T_{x_1}} - C_{D_1})\frac{2}{U_1} < 0$$

In the steady state, the following must be satisfied:

$$C_{T_{x_1}} - C_{D_1} = 0$$

$$(C_{T_{x_u}} - C_{D_u}) < 0$$

Criterios Estabilidad Estática - II

SIDE SPEED STABILITY

$$\frac{\partial(F_{A_y} + F_{T_y})}{\partial v} < 0$$

In the stability axis system:

$$F_{A_y} + F_{T_y} = (-C_y + C_{T_y})\bar{q}S$$

$$C_{y_\beta} + C_{T_{y_\beta}} < 0 \qquad \text{approximation } C_{T_{y_\beta}} \approx 0 \qquad C_{y_\beta} < 0$$

Criterios Estabilidad Estática - III

VERTICAL SPEED STABILITY

$$\frac{\partial(F_{A_z} + F_{T_z})}{\partial w} < 0$$

In the stability axis system:

$$F_{A_z} + F_{T_z} = (-C_L + C_{T_z})\overline{q}S$$

$$w = \alpha U_1$$

$$\frac{1}{U_1}(-C_{L_\alpha} + C_{T_{z_\alpha}})\overline{q}S < 0$$

$$C_{T_{z_\alpha}} \ll C_{L_\alpha}$$

$$C_{L_\alpha} > 0$$

Criterios Estabilidad Estática - IV

ANGLE OF ATTACK STABILITY

$$\frac{\partial(M_A + M_T)}{\partial\alpha} < 0$$

In the stability axis system:

$$M_A + M_T = (C_m + C_{m_T})\bar{q}S\bar{c}$$

$$C_{m_\alpha} + C_{m_{T_\alpha}} < 0$$

$C_{m_{T_\alpha}}$ is negligible compared with C_{m_α}

$$C_{m_\alpha} < 0$$

Criterios Estabilidad Estática - V

ANGLE OF SIDESLIP STABILITY

$$\frac{\partial(N_A + N_T)}{\partial\beta} > 0$$

In the stability axis system:

$$N_A + N_T = (C_n + C_{T_n})\overline{q}Sb$$

$$C_{n_\beta} + C_{n_{T_\beta}} > 0$$

$$C_{n_{T_\beta}} \ll C_{n_\beta}$$

$$C_{n_\beta} > 0 \qquad (C_{n_\beta})_{\beta \neq 0} > 0$$

Criterios Estabilidad Estática - VI

ROLL RATE STABILITY

$$\frac{\partial(L_A + L_T)}{\partial p} < 0$$

In the stability axis system:

$$L_A + L_T = (C_l + C_{l_T})\bar{q}Sb$$

$$C_{l_p} < 0$$

C_{l_p} is recognized as the roll damping derivative.

Criterios Estabilidad Estática - VII

PITCH RATE STABILITY

$$\frac{\partial(M_A + M_T)}{\partial q} < 0$$

In the stability axis system:

$$M_A + M_T = (C_m + C_{m_T})\bar{q}S\bar{c}$$

Neglecting the effect of thrust, $C_{m_q} < 0$

C_{m_q} is the pitch damping derivative

Criterios Estabilidad Estática - VIII

EFFECT OF FORWARD SPEED ON PITCHING MOMENT

$$\frac{\partial(M_A + M_T)}{\partial u} > 0$$

In the stability axis system:

$$M_A + M_T = (C_m + C_{m_T})\bar{q}S\bar{c}$$

$$(C_{m_u} + C_{m_{T_u}}) + (C_{m_1} + C_{m_{T_1}})\frac{2}{U_1} > 0$$

in steady state flight $(C_{m_1} + C_{m_{T_1}} = 0)$

$$(C_{m_u} + C_{m_{T_u}}) > 0$$

thrust contribution can be neglected

$$C_{m_u} > 0$$

C_{m_u} is the so–called tuck derivative

Criterios Estabilidad Estática - IX

EFFECT OF SIDESLIP ON ROLLING MOMENT

$$\frac{\partial(L_A + L_T)}{\partial\beta} < 0$$

In the stability axis system:

$$L_A + L_T = (C_l + C_{l_T})\bar{q}Sb$$

Neglecting the effect of thrust,

$$C_{l_\beta} < 0$$

C_{l_β} is also known as the airplane dihedral effect.

Criterios Estabilidad Estática - X

YAW RATE STABILITY

$$\frac{\partial(N_A + N_T)}{\partial r} < 0$$

$$N_A + N_T = (C_n + C_{n_T})\bar{q}Sb$$

Neglecting the effect of thrust, $C_{n_r} < 0$

C_{n_r} is the yaw damping derivative

Cooper-Harper Pilot Rating Scale

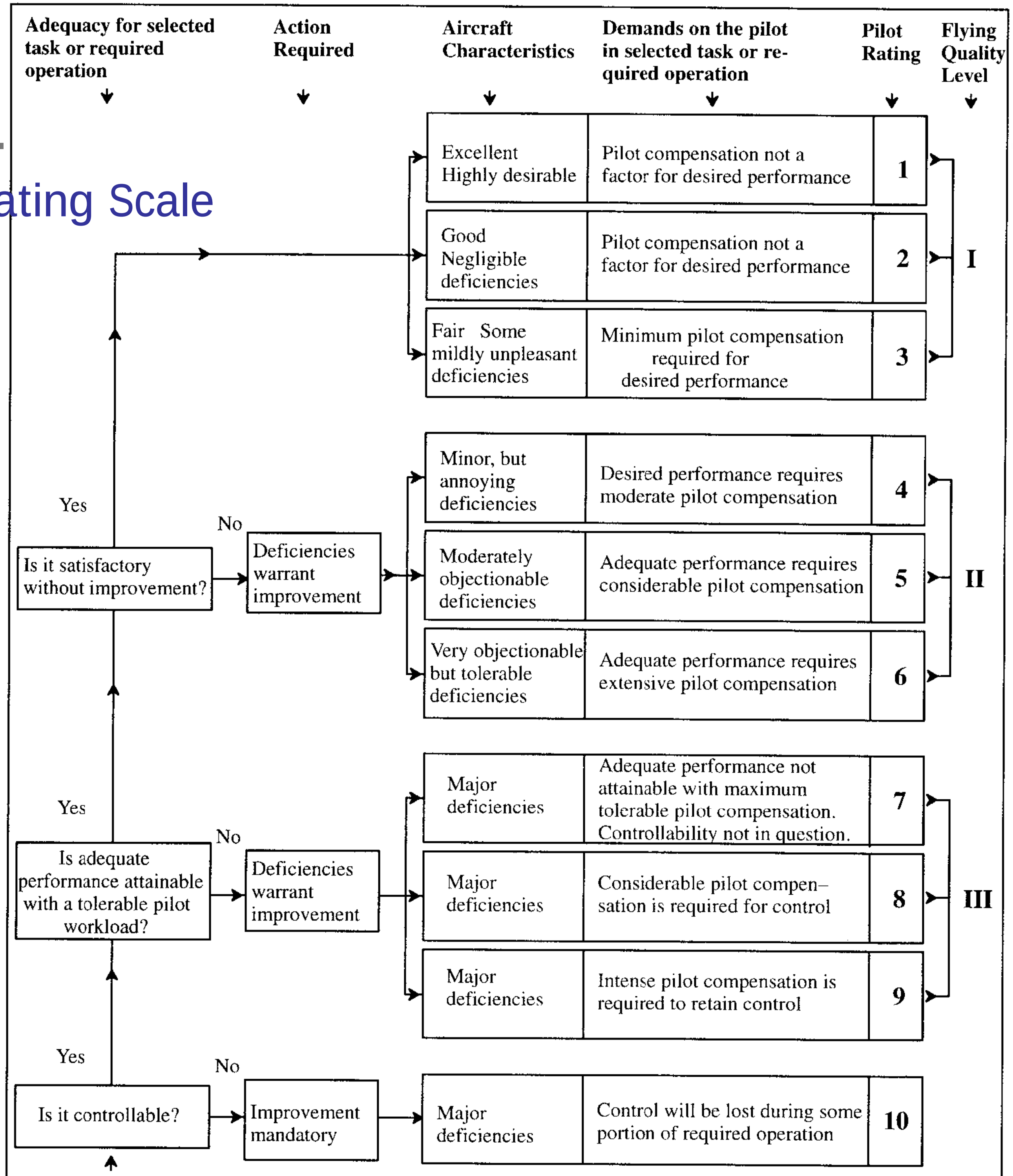


Figure 6.2 Cooper–Harper Pilot Rating Scale

Airplane Clases

Table 6.1 Definition of Airplane Classes			
MIL–F–8785C	**Examples**	**Civilian Equivalent**	**Examples**
Class I **Small, light airplanes such as:** * Light utility * Primary trainer * Light observation	* Cessna T–41 * Beech T–34C * Rockwell OV–10A	Very Light Aircraft (VLA) and FAR 23 category airplanes	* Cessna 210 * Piper Tomahawk * Edgeley Optica
Class II **Medium weight, low–to–medium maneuverability airplanes such as:** * Heavy utility / search and rescue * Light or medium transport / cargo / tanker * Early warning / electronic counter–measures / airborne command, control or communications relay * Anti–submarine * Assault transport * Reconnaissance * Tactical Bomber * Heavy Attack * Trainer for Class II	* Fairchild C–26A/B * Fairchild C–123 * Grumman E–2C * Boeing E–3A * Lockheed S–3A * Lockheed C–130 * Fairchild OA–10 * Douglas B–60 * Grumman A–6 * Beech T–1A	FAR 25 category airplanes	* Boeing 737, * Airbus A 320 * McDD MD–80
Class III **Large, heavy, low–to–medium maneuverability airplanes such as:** * Heavy transport / cargo / tanker * Heavy bomber * Patrol / early warning / electronic counter–measures / airborne command, control or communications relay * Trainer for Class III	* McDD C–17 * Boeing B–52H * Lockheed P–3 * Boeing E–3D * Boeing TC–135	FAR 25 category airplanes	* Boeing 747, * Airbus 340, * McDD MD–11
Class IV **High maneuverability airplanes such as:** * Fighter / interceptor * Attack * Tactical reconnaissance * Observation * Trainer for Class IV	* Lockheed F–22 * McDD F–15E * McDD RF–4 * Lockheed SR–71 * Northrop T–38	FAR 23 aerobatic category airplanes	* Pitts Special, * Sukhoi Su–26M

Categorias de Vuelo – MIL-F-8785C

Non-Terminal Flight Fases

Category A: Those non–terminal flight phases that require rapid maneuvering, precision tracking or precise flight path control. Included in this category are:

a) Air–to–air combat (CO)	None
b) Ground attack (GA)	None
c) Weapon delivery/launch (WD)	None
d) Aerial recovery (AR)	None
e) Reconnaissance (RC)	Observation, Pipeline spotting and monitoring
f) In–flight refuelling (receiver) (RR)	None as yet
g) Terrain following (TF)	None
h) Anti–submarine search (AS)	Fish spotting
i) Close formation flying (FF)	Air–show demonstrations

Category B: Those non–terminal flight phases that are normally accomplished using gradual maneuvers and without precision tracking, although accurate flight–path control may be required. Included in this category are:

a) Climb (CL)	Various climb segments
b) Cruise (CR)	Various cruise segments
c) Loiter (LO)	Flight in holding pattern
d) In–flight refuelling (tanker) (RT)	None as yet
e) Descent	Various descent segments
f) Emergency descent (ED)	Emergency descent
g) Emergency deceleration (DE)	None
h) Aerial delivery (AD)	Parachute drop

Categorias de Vuelo – MIL-F-8785C

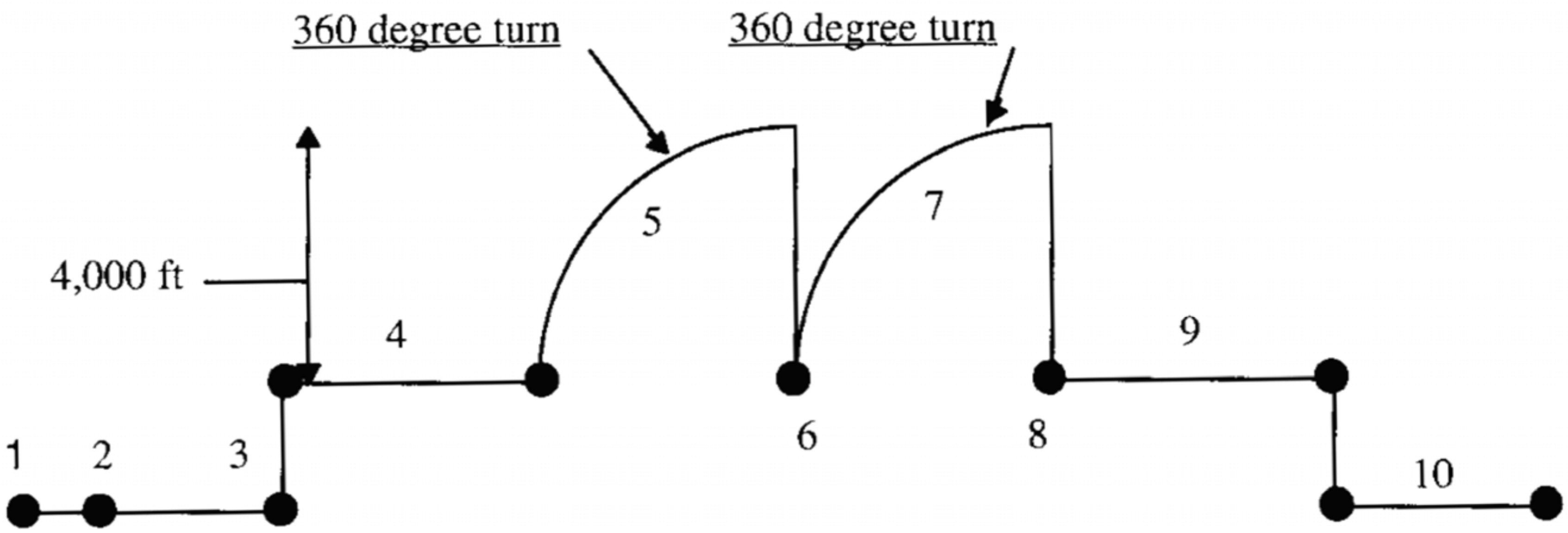
Terminal Flight Fases

Category C: Terminal flight phases are normally accomplished using gradual maneuvers and usually require accurate flight path control. Included in this category are:

a) Takeoff (TO)	Various takeoff segments
b) Catapult takeoff (CT)	None
c) Approach (PA)	Various approach segments
d) Wave–off / go–around (WO)	Aborted approach
e) Landing (L)	Various landing segments

Mission Profile - I

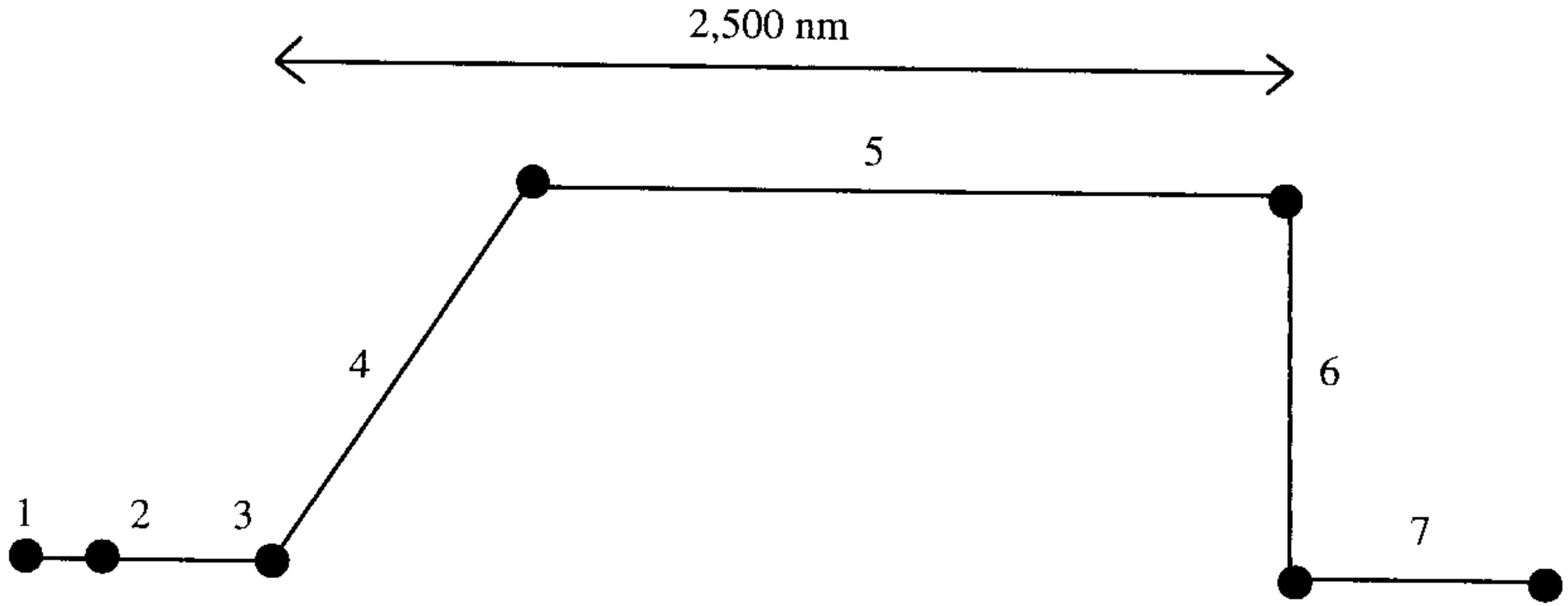
Mission Profile: Attack Airplane



1) Engine start and warm–up
2) Taxi
3) Takeoff and accelerate to 350 kts at sea–level
4) Dash 200 nm at 350 kts
5) 360 degree, sustained, 4.5g turn, including a 4,000 ft altitude gain
6) Release 2 bombs and fire 50% ammo
7) 360 degree, sustained, 4.5g turn, including a 4,000 ft altitude gain
8) Release 2 bombs and fire 50% ammo
9) Dash 200 nm at 350 kts
10) Landing, taxi, shutdown (no res.)

Mission Profile - II

Mission Profile: Passenger Transport



1) Engine start and warm–up
2) Taxi
3) Takeoff
4) Climb to 45,000 ft
5) Cruise
6) Descent
7) Landing, taxi, shutdown

Figure 6.3 Examples of Mission Profiles and Flight Phases for a Military and a Civilian Airplane

Level 1:	Flying qualities clearly adequate for the mission Flight Phase
Level 2:	Flying qualities adequate to accomplish the mission Flight Phase, but some increase in pilot workload or degradation in mission effectiveness, or both, exists.
Level 3:	Flying qualities such that the airplane can be controlled safely, but pilot workload is excessive or mission effectiveness is inadequate, or both. Category A Flight Phases can be terminated safely, and Category B and C Flight Phases can be completed.

Criterios estabilidad dinámica longitudinal

Table 6.7 Phugoid Damping Requirements

	MIL–F–8785C	VLA, FAR 23 and FAR 25
Level I:	$\zeta_{ph} \geq 0.04$	No requirement
Level II:	$\zeta_{ph} \geq 0$	No requirement
Level III:	$T_{2_{ph}} \geq 55$ sec	No requirement

Table 6.9 Short Period Damping Ratio Limits
MIL–F–8785C

	Category A and C Flight Phases			Category B Flight Phases		
Level	Minimum		Maximum	Minimum		Maximum
Level 1*	0.35	$\leftarrow \zeta_{sp} \rightarrow$	1.30	0.30	$\leftarrow \zeta_{sp} \rightarrow$	2.00
Level 2	0.25	$\leftarrow \zeta_{sp} \rightarrow$	2.00	0.20	$\leftarrow \zeta_{sp} \rightarrow$	2.00
Level 3	0.15 * *	$\leftarrow \zeta_{sp} \rightarrow$	no maximum	0.15 *	$\leftarrow \zeta_{sp} \rightarrow$	no maximum

* For VLA, FAR 23 and FAR 25 : ζ_{sp} must be heavily damped

** For altitudes above 20,000 ft this requirement may be reduced if approved by the procuring activity

Criterios estabilidad dinámica lateral-direccional

Table 6.12 Minimum Dutch Roll Undamped Natural Frequency and Damping Ratio Requirements

Mil-F-8785C

Level	Flight Phase Category	Airplane Class	Min. ζ_d *	Min. $\zeta_d\omega_{n_d}$ * rad/sec	Min. ω_{n_d} rad/sec
Level 1	A (Combat and Ground Attack)	IV	0.4	-	1.0
	A (Other)	I and IV	0.19	0.35	1.0
		II and III	0.19	0.35	0.4**
	B	All	0.08	0.15	0.4**
	C	I, II–C and IV	0.08	0.15	1.0
		II–L and III	0.08	0.10	0.4**
Level 2	All	All	0.02	0.05	0.4**
Level 3	All	All	0	-	0.4**

* The governing requirement is that which yields the largest value of ζ_d.
Note : For Class III $\zeta_d = 0.7$ is the maximum value required.
* * Class III airplanes may be excepted from these requirements, subject specific approval.

Civilian Requirements:

FAR 23 and VLA: $\zeta_d > 0.052$ with controls – free and controls – fixed

FAR 25: $\zeta_d > 0$ with controls – free and must be controllable without exceptional pilot skills

Criterios estabilidad dinámica lateral-direccional

Table 6.13 Minimum Time to Double the Amplitude in the Spiral Mode
MIL-F-8785C

Flight Phase Category	Level 1	Level 2	Level 3
A and C	$T_{2_s} > 12$ sec	$T_{2_s} > 8$ sec	$T_{2_s} > 4$ sec
B	$T_{2_s} > 20$ sec	$T_{2_s} > 8$ sec	$T_{2_s} > 4$ sec

Civilian Requirements: None

Table 6.14 Maximum Allowable Roll Mode Time Constant
MIL-F-8785C

Flight Phase Category	Airplane Class	Level 1	Level 2	Level 3
A	I and IV	$T_r \leq 1.0$ sec	$T_r \leq 1.4$ sec	$T_r \leq 10.0$ sec
	II and III	$T_r \leq 1.4$ sec	$T_r \leq 3.0$ sec	$T_r \leq 10.0$ sec
B	All	$T_r \leq 1.4$ sec	$T_r \leq 3.0$ sec	$T_r \leq 10.0$ sec
C	I, II-C and IV	$T_r \leq 1.0$ sec	$T_r \leq 1.4$ sec	$T_r \leq 10.0$ sec
	II-L and III	$T_r \leq 1.4$ sec	$T_r \leq 3.0$ sec	$T_r \leq 10.0$ sec

Civilian Requirements: None

Roll Effectiveness Requirements - I

$$\phi = 60\,\text{deg} \;\; t \;\; \phi = 0\,\text{deg}$$

Military Airplanes MIL-F-8785C

Tiempo máximo (segs) que puede tardar en realizar un bank angle de 0º a 60º

		Flight Phase Category					
		A		B		C	
Airplane Class	Level	$\phi = 60$ deg t $\phi = 0$ deg	$\phi = 45$ deg t $\phi = 0$ deg	$\phi = 60$ deg t $\phi = 0$ deg	$\phi = 45$ deg t $\phi = 0$ deg	$\phi = 30$ deg t $\phi = 0$ deg	$\phi = 25$ deg t $\phi = 0$ deg
I	1	1.3	–	1.7	–	1.3	–
I	2	1.7	–	2.5	–	1.8	–
I	3	2.6	–	3.4	–	2.6	–
II–L	1	–	1.4	–	1.9	1.8	–
II–L	2	–	1.9	–	2.8	2.5	–
II–L	3	–	2.8	–	3.8	3.6	–
II–C	1	–	1.4	–	1.9	–	1.0
II–C	2	–	1.9	–	2.8	–	1.5
II–C	3	–	2.8	–	3.8	–	2.0

Low speed range represents takeoff and approach speeds
Medium speed range represents speeds up to 70% of maximum level speed
High speed range represents speeds from 70% to 100% of maximum level speed

Roll Effectiveness Requirements - II

Military Airplanes MIL-F-8785C

$$t_{\phi=0\,\text{deg}}^{\phi=60\,\text{deg}}$$

Tiempo máximo (segs) que puede tardar en realizar un bank angle de 0º a 60º

Class III		Flight Phase Category		
		A	B	C
Level	Speed Range *	$t_{\phi=0\,\text{deg}}^{\phi=30\,\text{deg}}$	$t_{\phi=0\,\text{deg}}^{\phi=30\,\text{deg}}$	$t_{\phi=0\,\text{deg}}^{\phi=30\,\text{deg}}$
1	Low	1.8	2.3	2.5
	Medium	1.5	2.0	2.5
	High	2.0	2.3	2.5
2	Low	2.4	3.9	4.0
	Medium	2.0	3.3	4.0
	High	2.5	3.9	4.0
3	All	3.0	5.0	6.0

Roll Effectiveness Requirements - III

Table 6.16 Roll Effectiveness Requirements for Class IV Airplanes

MIL–F–8785C: **NOTE: All times, t in seconds**

		Flight Phase Category				
		A			B	C
Level	Speed Range *	$\phi = 30$ deg t $\phi = 0$ deg	$\phi = 50$ deg t $\phi = 0$ deg	$\phi = 90$ deg t $\phi = 0$ deg	$\phi = 90$ deg t $\phi = 0$ deg	$\phi = 30$ deg t $\phi = 0$ deg
1	Very Low	1.1	–	–	2.0	1.1
	Low	1.1	–	–	1.7	1.1
	Medium	–	–	1.3	1.7	1.1
	High	–	1.1	–	1.7	1.1
2	Very Low	1.6	–	–	2.8	1.3
	Low	1.5	–	–	2.5	1.3
	Medium	–	–	1.7	2.5	1.3
	High	–	1.3	–	2.5	1.3
3	Very Low	2.6	–	–	3.7	2.0
	Low	2.0	–	–	3.4	2.0
	Medium	–	–	2.6	3.4	2.0
	High	–	2.6	–	3.4	2.0

Roll Effectiveness Requirements

Civil Airplanes

$$\phi = 60\,\text{deg} \quad t \quad \phi = 0\,\text{deg}$$

Tiempo máximo (segs) que puede tardar en realizar un bank angle de 0º a 60º

Table 6.17 Roll Effectiveness Requirements for Civilian Airplanes

Note: All times, t in seconds			VLA	FAR 23	FAR 25
Flight Phase	Speed	Weight (lbs)	$\phi = +30\,\text{deg}$ t $\phi = -30\,\text{deg}$	$\phi = +30\,\text{deg}$ t $\phi = -30\,\text{deg}$	
Takeoff	$1.2V_{S_{TO}}$	$W \leq 6{,}000$	5	5	No requirement
		$W > 6{,}000$	Not applicable	$t = \frac{W + 500}{1{,}300}$	No requirement
Landing	$1.3V_{S_{PA}}$	$W \leq 6{,}000$	4	4	No requirement
		$W > 6{,}000$	Not applicable	$t = \frac{W + 2{,}800}{2{,}200}$	No requirement

Note: For FAR 25 it is suggested to use the Class II or Class III military requirements

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